1.2 FUNCTIONS AND THEIR PROPERTIES
(Part 2 - Asymptotes)

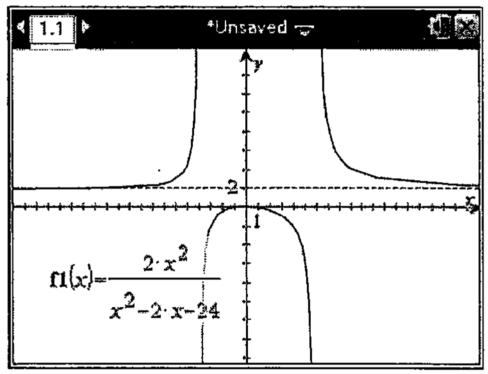
Horizontal Asymptotes

The line $y = b$ is a **horizontal** asymptote of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of b as x approaches $+\infty$ or $-\infty$.

$$\lim_{x \to -\infty} f(x) = b \quad \text{or} \quad \lim_{x \to +\infty} f(x) = b$$

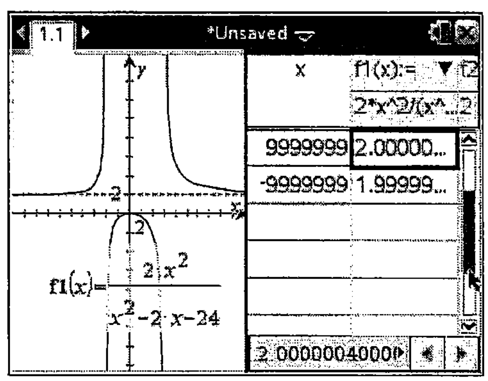
Example 1: Identify any horizontal asymptote of $f(x) = \dfrac{2x^2}{x^2 - 2x - 24}$.

$$\lim_{x \to \pm\infty} \frac{2x^2}{x^2 - 2x - 24} =$$



Equation of horizontal asymptote ______________________________

Check numerically:

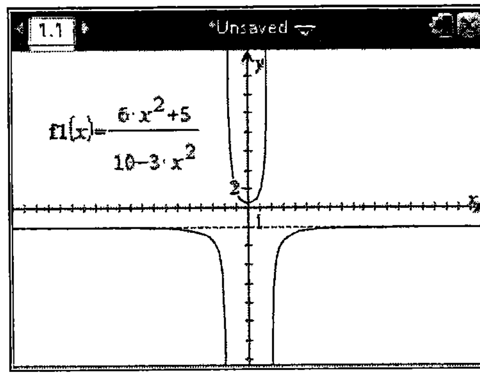


$\lim_{x \to -\infty} f(x) =$ ______

$\lim_{x \to +\infty} f(x) =$ ______

Example 2: Identify any horizontal asymptote of . $f(x)=\dfrac{6x^2+5}{10-3x^2}$

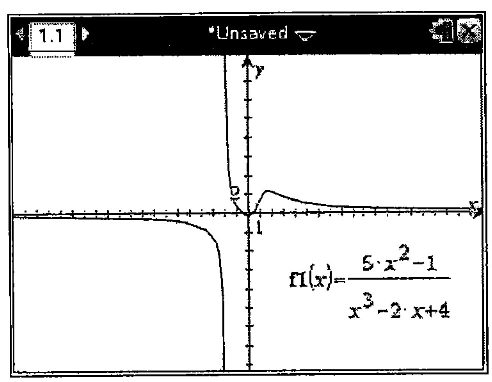
$$\lim_{x\to\pm\infty}\frac{6x^2+5}{10-3x^2}=$$



Equation of horizontal asymptote ______________________________

Example 3: Identify any horizontal asymptote of . $f(x)=\dfrac{5x^2-1}{x^3-2x+4}$

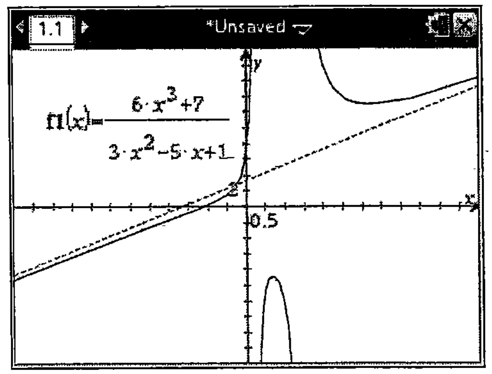
$$\lim_{x\to\pm\infty}\frac{5x^2-1}{x^3-2x+4}=$$



Equation of horizontal asymptote ______________________________

Example 4: Identify any horizontal asymptote of . $f(x)=\dfrac{6x^3+7}{3x^2-5x+1}$

$$\lim_{x\to\infty}\frac{6x^3+7}{3x^2-5x+1}=$$



$$\lim_{x\to-\infty}\frac{6x^3+7}{3x^2-5x+1}=$$

End Behavior of Rational Functions

It is helpful to know how a function behaves as it goes off toward either "end" of the x-axis.

Let $f(x) = \dfrac{ax^n + \ldots}{cx^k + \ldots}$ be a rational function whose numerator has degree n and whose denominator has degree k.

- If $n < k$, then the horizontal asymptote is the x-axis ($y = 0$)
- If $n = k$, then the horizontal asymptote is the line $y = \dfrac{a}{c}$.
- If $n > k$, then the quotient polynomial when the numerator is divided by the denominator is the asymptote that describes the end behavior of the graph.

Example 1: $f(x) = \dfrac{3x - 6}{5 - 2x}$

Asymptote ______________________

Example 2: $f(x) = \dfrac{x^3 - x^2 - x - 1}{x^5 - 36x}$

Asymptote ______________________

Example 3: $f(x) = \dfrac{2x^2 + 3x - 12}{x + 4}$

Asymptote ______________________

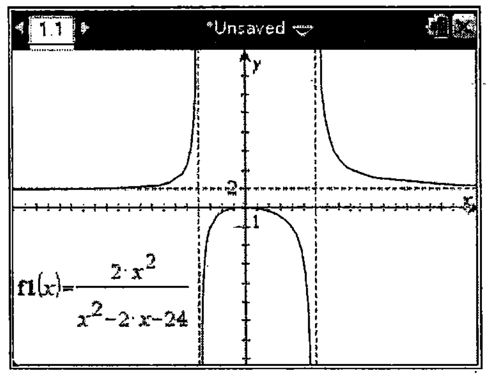
Vertical Asymptotes 1.2

The line $x = a$ is a **vertical** asymptote of the graph of a function $y = f(x)$ if $f(x)$ approaches a limit of $+\infty$ or $-\infty$ as x approaches a from either direction.

$$\lim_{x \to a^-} f(x) = \pm\infty \quad \text{or} \quad \lim_{x \to a^+} f(x) = \pm\infty$$

Example 1: Identify any vertical asymptotes of $f(x) = \dfrac{2x^2}{x^2 - 2x - 24}$.

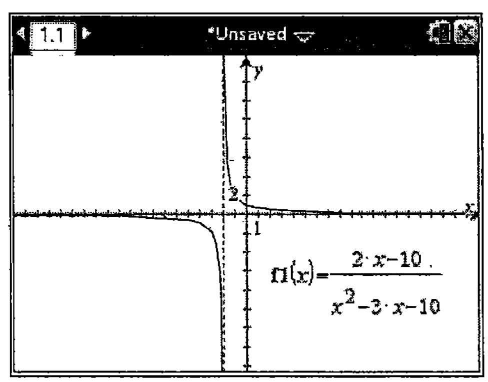
$$\frac{2x^2}{x^2 - 2x - 24} = \frac{2x^2}{(\qquad)(\qquad)}$$



Equations of vertical asymptotes ______________________________

Example 2: Identify any vertical asymptotes of. $f(x) = \dfrac{2x - 10}{x^2 - 3x - 10}$

$$\frac{2x - 10}{x^2 - 3x - 10} = \frac{2(\qquad)}{(\qquad)(\qquad)}$$



Equation of vertical asymptote ______________________________

Example 3:

Identify any vertical asymptotes of. $f(x)=\dfrac{-2x^2+4x+16}{x^2+x-12}$.

$\dfrac{-2x^2+4x+16}{x^2+x-12}=$ ______________

Equations of vertical asymptotes ____________________________

Identify any horizontal asymptote of $f(x)=\dfrac{-2x^2+4x+16}{x^2+x-12}$.

Equation of horizontal asymptote ____________________________