2.6 Graphs of Reciprocal Functions

V.A. are found in denominator

H.A. comparing n / d degree

- $n > d$ no HA
- $n = d$ use coefficients
- $n < d$ $y = 0$

Ex 1 $f(x) = \frac{1}{x-2}$

VA $x = 2$

Domain: $(-\infty, 2) \cup (2, \infty)$

$\lim_{x \to 2^-} f(x) = -\infty$

$\lim_{x \to 2^+} f(x) = \infty$

HA $y = 0$

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

Ex 2

a) $f(x) = \frac{2}{x+3}$

$= 2\left(\frac{1}{x+3}\right)$

$\frac{1}{x}$

left 3 units

V Stretch 2

b) $h(x) = \frac{3x-7}{x-2}$

long divide

$h(x) = 3 - \frac{1}{x-2}$

$-\frac{1}{x-2} + 3$

$-\left(\frac{1}{x-2}\right) + 3$

Reflect x-axis

Right 2

Up 3

VA $x = 2$

$\lim_{x \to 2^+} h(x) = -\infty$

$\lim_{x \to 2^-} h(x) = \infty$

HA $y = 3$

$\lim_{x \to -\infty} h(x) = 3$

$\lim_{x \to \infty} h(x) = 3$

Ex 3

$$f(x) = \frac{x^2+2}{x^2+1}$$

VA none

HA $y = 1$

graph

$\lim_{x\to\infty} f(x) = 1$

$\lim_{x\to-\infty} f(x) = 1$

algebraically

long divide

$$f(x) = \frac{1}{x^2+1} + 1$$

$y=0$ but then shifts up 1

$$\frac{x^2+2}{x^2+1}$$

$$x^2+1 \overline{)\, x^2+2}$$

$$\begin{array}{r} 1 \\ -x^2+1 \\ \hline 1 \end{array}$$

$$1 + \frac{1}{x^2+1}$$

Ex 4

$$f(x) = \frac{x^3}{x^2-9} = \frac{x^3}{(x+3)(x-3)}$$

No HA

VA $x = \pm 3$

long division

$$\begin{array}{r} x \\ x^2-9 \overline{)\, x^3 + 0} \\ -x^3 - 9x \\ \hline 9x \end{array}$$

$$f(x) = x + \frac{9x}{x^2-9}$$

b/c not a constant a slant asymptote occurs

p245

#2,4,6,7,9,11-14

20,21,23,26,29