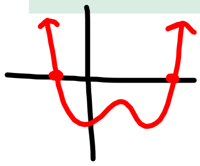
2.5 Complex Zeros and the Fundamental Theorem of Algebra

THEOREM Fundamental Theorem of Algebra

A polynomial function of degree n has n complex zeros (real and nonreal). Some of these zeros may be repeated.



Deg: 4
2 real zeros

Ex 1 Exploring Fundamental Polynomial Connections

$i = \sqrt{-1}$

Write the polynomial function in standard form, identify the zeros of the functions and x intercepts of its graph.

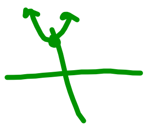
$i^2 = -1$

real zeros

(a) $f(x) = (x - 2i)(x + 2i)$

$x^2 + \cancel{2xi} - \cancel{2xi} - \cancel{4i^2}$ $+4$

$f(x) = x^2 + 4$



zeros: $\pm 2i$

x-int: NO

(b) $f(x) = (x - 5)(x - \sqrt{2}i)(x + \sqrt{2}i)$

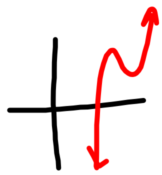
$(x - i\sqrt{2})(x + i\sqrt{2})$

$(x-5)(x^2 - \cancel{2i^2})$ $+2$

$f(x) = x^3 - 5x^2 + 2x - 10$

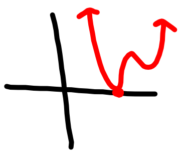
zeros: $x = 5, \pm i\sqrt{2}$

x-int: $x = 5$



(c) $f(x) = (x - 3)(x - 3)(x - i)(x + i)$

$(x^2 - 6x + 9)(x^2 - \cancel{i^2})$ $+1$



$f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9$

zeros: $x = 3$, w/ mult of 2, $\pm i$

x-int: $x = 3$

THEOREM Complex Conjugate Zeros

Suppose that $f(x)$ is a polynomial function with *real coefficients*. If a and b are real numbers with $b \neq 0$ and $a + bi$ is a zero of $f(x)$, then its complex conjugate $a - bi$ is also a zero of $f(x)$.

* Imaginary zeros are in pairs

Ex 2 Finding a Polynomial Given Zeros

Write a polynomial function with a minimum degree in standard form with real coefficients whose zeros include -3, 4, and 2 - i $2+i$

$$f(x) = (x+3)(x-4)(x-(2-i))(x-(2+i))$$

$$(x+3)(x-4)(x-2+i)(x-2-i)$$

$$x^2 - 2x - x\cancel{i} - 2x + \cancel{xi} + 4 + 2\cancel{i} - \cancel{2i} - i^2 \quad +1$$

$$(x^2 - x - 12)(x^2 - 4x + 5)$$

$$f(x) = x^4 - 5x^3 - 3x^2 + 43x - 60$$

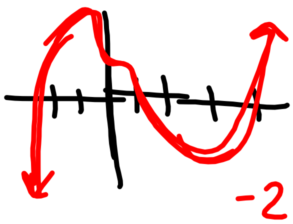
Ex 3 Finding a Polynomial Given Zeros

Write a polynomial function with a minimum degree in standard form with real coefficients whose zeros include x = 1, x = 1 + 2i, and x = 1 - i

Ex 4 Factoring Polynomial with Complex Zeros

Find all the zeros of $f(x) = x^5 - 3x^4 - 5x^3 + 5x^2 - 6x + 8$

(peek at graph)



x-intercepts: $-2, 1, 4$

$(x+2)(x-1)(x-4)(x^2+1)$

$(x-i)(x+i)$

$$\begin{array}{r|rrrrrr} -2 & 1 & -3 & -5 & 5 & -6 & 8 \\ & \downarrow & -2 & 10 & -10 & 10 & -8 \\ \hline 1 & 1 & -5 & 5 & -5 & 4 & 0 \\ & \downarrow & 1 & -4 & 1 & -4 & \\ \hline 4 & 1 & -4 & 1 & -4 & 0 & \\ & \downarrow & 4 & 0 & 4 & & \\ \hline & 1 & 0 & 1 & 0 & & \end{array}$$

Zeros: $-2, 1, 4, \pm i$

Ex 5 Finding Complex Zeros

The complex number $z = 1 - 2i$ is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$ the remaining zeros of $f(x)$, and write it in its linear factorization.

$x = 1-2i \qquad x = 1+2i$

$(x-(1-2i))(x-(1+2i))$

$\underbrace{(x-1+2i)(x-1-2i)}$

synthetic can be messy w/ complex zeros

Use the pair to create a poly. to Long Divide

$$\begin{array}{r} 4x^2+8x+13 \\ x^2-2x+5 \overline{\big) 4x^4+0x^3+17x^2+14x+65} \end{array}$$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\dfrac{-8 \pm \sqrt{8^2-4(4)(13)}}{2(4)}$

$\dfrac{-8 \pm \sqrt{64-208}}{8}$

$\dfrac{-8 \pm \sqrt{-144}}{8}$

$\dfrac{-8 \pm 12i}{8} = -1 \pm \frac{3}{2}i$

$(x-1+2i)(x-1-2i)(4x^2+8x+13)$

$(x-1+2i)(x-1-2i)(x-(-1+\frac{3}{2}i))(x-(-1-\frac{3}{2}i))$

$(x-1+2i)(x-1-2i)(x+1-\frac{3}{2}i)(x+1+\frac{3}{2}i)$

p234 # 2, 3, 6, 10, 12, 14, 19, 27, 28, 30, 33, 35