Upper and Lower Bound Tests for Real Zeros

Let f be a polynomial function of degree $n \geq 1$ with a positive leading coefficient. Suppose $f(x)$ is divided by $x - k$ using synthetic division.

- If $k \geq 0$ and every number in the last line is nonnegative (positive or zero), then k is an *upper bound* for the real zeros of f.
- If $k \leq 0$ and the numbers in the last line are alternately nonnegative and non-positive, then k is a *lower bound* for the real zeros of f.

K+ last line of synthetic div is nonnegative (UB)

K- last line of syn div alternates +/- (LB)

Ex 6 Establishing Bounds for Real Zeros

Prove that all the real zeros of $2x^4 - 7x^3 - 8x^2 + 14x + 8$ must lie in the interval $[-2, 5]$

$$\begin{array}{r|rrrrr} 5 & 2 & -7 & -8 & 14 & 8 \\ \hline & 2 & 3 & 7 & 49 & 253 \end{array}$$

Yes 5 is an UB b/c all # are nonnegative

$$\begin{array}{r|rrrrr} -2 & 2 & -7 & -8 & 14 & 8 \\ \hline & 2 & -11 & 14 & -14 & 36 \end{array}$$

Yes -2 is a LB b/c #'s alternate +/-

Ex 7 Finding the Real Zeros

Find the real zeros of $f(x) = 2x^4 - 7x^3 - 8x^2 + 14x + 8$

$\dfrac{8}{2}\ \dfrac{\pm1, \pm8, \pm2, \pm4}{\pm1, \pm2}$

combos $\pm1, \pm2, \pm4, \cancel{\pm8}, \pm\frac{1}{2}$,

Remember
LB -2
UB 5

Pick one to try

$$\begin{array}{r|rrrrr} 4 & 2 & -7 & -8 & 14 & 8 \\ & & 8 & 4 & -16 & -8 \\ \hline & 2 & 1 & -4 & -2 & 0 \end{array}$$

$(x-4)(2x^3+1x-4x-2)$

$$\begin{array}{r|rrrr} -\frac{1}{2} & 2 & 1 & -4 & -2 \\ & & -1 & 0 & 2 \\ \hline & 2 & 0 & -4 & 0 \end{array}$$

$(x-4)(2x+1)(2x^2-4)$
$2(x^2-2)$

$x^2 - 2 = 0$
$x^2 = 2$
$x = \pm\sqrt{2}$

$2(x-4)(2x+1)(x+\sqrt{2})(x-\sqrt{2})$