dividend ÷ divisor

quotient + remainder

CHECK

divisor • quotient

5) $\dfrac{x^4-2x^3+3x^2-4x+6}{x^2+2x-1}$

$x^2-4x+12+\dfrac{-32x+18}{x^2+2x-1}$

$$
\begin{array}{r l}
 & x^2-4x+12 \\
x^2+2x-1 \,\big) & x^4-2x^3+3x^2-4x+6 \\
- & x^4+2x^3-1x^2 \\
\hline
 & -4x^3+4x^2-4x \\
- & -4x^3-8x^2+4x \\
\hline
 & 12x^2-8x+6 \\
- & 12x^2+24x-12 \\
\hline
 & -32x+18
\end{array}
$$

CHECK

$\overset{Q}{(x^2-4x+12)}\overset{D}{(x^2+2x-1)}+\overset{R}{(-32x+18)}$

19) $(x^3-x^2+x-1)\div(x-1)$

$$
\begin{array}{c|cccc}
1 & 1 & -1 & 1 & -1 \\
 & \downarrow & 1 & 0 & 1 \\
\hline
 & 1 & 0 & 1 & 0 \\
\end{array}
$$

$\dfrac{x^5}{x}$

x^2+0x+1 $\quad$ $\underset{R}{0}$

x^2+1

b/c $r=0$

$x-1$ is a factor

$f(1)=(1)^3-(1)^2+(1)-1$

$\cancel{1-1}+\cancel{1-1}$

0

17) $f(x)=2x^3-3x^2+4x-7$

$f(2)=2(2)^3-3(2)^2+4(2)-7$

$f(2)=5$

$x-2$ a factor?

NO

THEOREM Rational Zeros Theorem

Suppose f is a polynomial function of degree $n \geq 1$ of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0,$$

with every coefficient an integer and $a_0 \neq 0$. If $x = p/q$ is a rational zero of f, where p and q have no common integer factors other than 1, then

- p is an integer factor of the constant coefficient a_0, and
- q is an integer factor of the leading coefficient a_n.

$2x^3 + 5x^2 - 4x + 6$

LC: q constant: p

Ex 4 Rational Zeros

Find the rational zeros of $f(x) = 1x^3 - 3x^2 + 1$

constant: 1
LC: 1

factors $\dfrac{\pm 1}{\pm 1}$

Combinations 1 or −1

CHECK Synthetic

$1 \,|\, 1 \quad -3 \quad 0 \quad 1$

−1

Remainder THM

$f(-1) = (-1)^3 - 3(-1)^2 + 1$

$f(-1) = -3$

Not factors

No Rational Roots

Ex 5 Rational Zeros

Find the rational zeros of $f(x) = 3x^3 + 4x^2 - 5x - 2$

$\dfrac{\text{factors of } -2: \ \pm 1, \pm 2}{\text{factors of } 3: \ \pm 1, \pm 3}$

$\pm 1, \pm 2, \pm \frac{1}{3}, \pm \frac{2}{3}$

- If calc available can take a peek to better guess which is a factor. Otherwise, guess & check

Try

1	3	4	−5	−2
	↓	3	7	2
	3	7	2	0

Since r=0, $\underline{x-1}$ is a factor

$(x-1)\underbrace{(3x^2+7x+2)}_{\text{factor}} = (x-1)(3x+1)(x+2)$

zeros @ $x = 1, -\frac{1}{3}, -2$