2.3 Polynomials with Higher Degrees

DEFINITION The Vocabulary of Polynomials

- Each monomial in this sum—$a_n x^n$, $a_{n-1}x^{n-1}$, . . . , a_0—is a **term** of the polynomial.
- A polynomial function written in this way, with terms in descending degree, is written in **standard form**.
- The constants a_n, a_{n-1}, . . . , a_0 are the **coefficients** of the polynomial.
- The term $a_n x^n$ is the **leading term**, and a_0 is the constant term.

$x^3 + x^2 - 4x - 1\cancel{x^0}$

Ex 1 Graph Transformation of a Monomial Function (describe)

$y = 4x^3$

(a) $g(x) = 4(x + 1)^3$

monomial function

$g(x) = 4x^3$

Shift left 1 unit

(b) $h(x) = -(x - 2)^4 + 5$

monomial function

$h(x) = -1x^4$

Shift right 2 units
up 5 units

THEOREM Local Extrema and Zeros of Polynomial Functions

A polynomial function of degree n has at most $n - 1$ local extrema and at most n zeros.

$y = x^2$ degree 2, 1 local extrema, At most 2 zeros

EXPLORATION 1 Investigating the End Behavior of $f(x) = a_n x^n$

Graph each function in the window $[-5, 5]$ by $[-15, 15]$. Describe the end behavior using $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.

1. **(a)** $f(x) = 2x^3$ **(b)** $f(x) = -x^3$
   (c) $f(x) = x^5$ **(d)** $f(x) = -0.5x^7$
2. **(a)** $f(x) = -3x^4$ **(b)** $f(x) = 0.6x^4$
   (c) $f(x) = 2x^6$ **(d)** $f(x) = -0.5x^2$
3. **(a)** $f(x) = -0.3x^5$ **(b)** $f(x) = -2x^2$
   (c) $f(x) = 3x^4$ **(d)** $f(x) = 2.5x^3$

1. (a) $\infty; -\infty$ (b) $-\infty; \infty$
   (c) $\infty; -\infty$ (d) $-\infty; \infty$
2. (a) $-\infty; -\infty$ (b) $\infty; \infty$
   (c) $\infty; \infty$ (d) $-\infty; -\infty$
3. (a) $-\infty; \infty$ (b) $-\infty; -\infty$
   (c) $\infty; \infty$ (d) $\infty; -\infty$

Describe the patterns you observe. In particular, how do the values of the coefficient a_n and the degree n affect the end behavior of $f(x) = a_n x^n$?

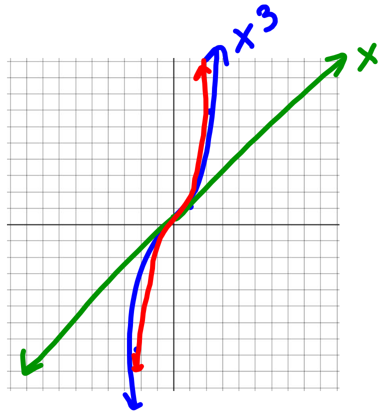
Ex2 Graphing Combinations of Monomials

a) $f(x) = x^3 + x$

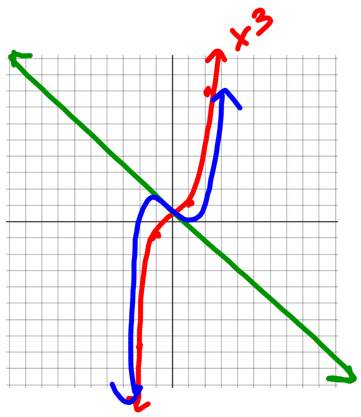
$y_1 = x^3$

$y_2 = x$

$y_3 = x^3 + x$ blends the two

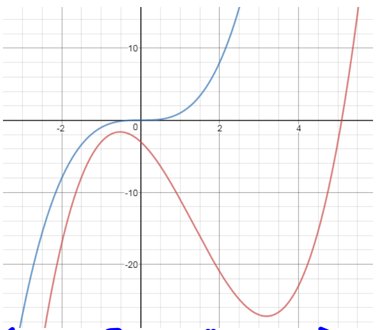


b) $g(x) = x^3 - x$



Ex 3

$f(x) = x^3 - 4x^2 - 5x - 3$ $g(x) = x^3$



$[-7, 7]$ by $[-25, 25]$

View as: ① $[-14, 14]$ by $[-200, 200]$

② $[-56, 56]$ by $[-12{,}800, 12800]$

If you zoom out enough the two functions should look almost identical

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